An Analysis on the Inter-Cell Station Dependency Probability in an IEEE 802.11 Infrastructure WLANs

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I. System Model

To ensure desired rate (say atleast r_t Mbps) of association to wireless devices in IEEE 802.11g infrastructure mode, we may have to deploy a dense layout of access points, with significant overlaps among their coverage regions. Let P_t be the power level required to ensure a target rate of at least $r_t \, Mbps$.

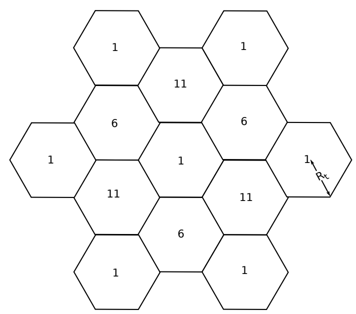

Fig. 1. Hexagonal Micro-cellular layout of IEEE 802.11g with cell radius R_t and 3 non-overlapping channels.

The cell radius R_t is related to the power level P_t as follows

$$P_t = S \cdot 10^{\frac{-\xi}{10}} \cdot \left(\frac{R_t}{R_0}\right)^{-\eta} \tag{1}$$

where S is the transmit power, R_0 is the "far field" *reference distance*, η is the path loss exponent and ξ is a Gaussian random variable with mean 0 and variance σ^2. Let r_{min} be the minimum transmission rate possible. Let P_{min} and R_{min} be the power level and distance at which r_{min} is sustainable. Then,

$$P_{min} = S \cdot 10^{\frac{-\xi}{10}} \cdot \left(\frac{R_{min}}{R_0}\right)^{-\eta} \tag{2}$$

Dividing Equation (2) by Equation (1) and rearranging the terms, we obtain

$$R_{min} = R_t \cdot \left(\frac{P_t}{P_{min}}\right)^{1/\eta}$$

Let R_{cs} and R_i be the carrier sensing and interference range of a node in a cell respectively. Let us assume that We assume $R_i = \alpha \cdot R_{cs}$, where $\alpha \geq 1$ which implies that any node within the R_{cs} of a receiver can cause interference and nodes outside the R_{cs} cannot. Thus, the interference region of a node is a disk of radius $R_{cs} = \gamma(\eta, P_t, P_{min}) \cdot R_t$ centred at the node itself, where $\gamma(\eta, P_t, P_{min}) = 2\alpha \cdot \left(\frac{P_t}{P_{min}}\right)^{1/\eta}$. For ease of analysis, we assume that the interference region is a regular hexagon inscribed with the disk of radius R_{cs}.

Given an association of stations with access points, we can think of each STA-AP association as a *link*. For two stations associated with two different access points, we say that the corresponding links are *dependent* if throughputs obtained on each link with simultaneous bulk TCP transfers are lower than the throughputs obtained when the transfers are performed on the links individually. Based on the above definition of *dependence*, it is easy to see that two stations associated with the same access point are *dependent*. This dependence relation between links is represented as an undirected graph called the *link dependence* graph, whose vertex set is the set of links.

In this document, we are primarily interested in computing the probabilities of various types of dependencies that can occur in a deployment as shown in Figure 1. Let us assume that we have are given a dependence graph $\mathcal{G}$. Recall that nodes in the dependence graph corresponds to link the actual network (a STA-AP association). Also, since each station can associate with only one access point at a time, each station will correspond to one and only one link in the dependence graph. Thus, we can correlate a link in the dependence graph to dependency between two stations. Consider two stations S_1 and S_2. Let the stations S_1 and S_2 be associated with access point A_1 and A_2 respectively. We classify the dependency between stations S_1 and S_2 into three main types as follows:

- ***Type I Dependence***: Stations S_1 and S_2 are within the interference range of each other. The stations are also outside the interference range of the each others access point and the access points do not interfere with each other.
- ***Type II Dependence***: Access points of stations S_1 and S_2 interfere with each other.
- ***Type III Dependence***: Access points of stations S_1 and S_2 do not interfere with each other. Station S_2 is within the interference range of access points A_1 and/or station S_1 is within the interference range of access points A_2.

Let $D_j = \nu_j \cdot R_t$ be the distance between the centres of the j^{th} tier co-channel cells. The maximum and minimum distance between two nodes in the j^{th} tier co-channel cells are $D_j + 2R_t$ and $D_j - 2R_t$ respectively. Thus, if stations S_1 and S_2 belong to cells whose centres are atleast $R_i + 2R_t$ units apart, the stations will not depend on each other. Let

$$j_0 = \max\{j \geq 0 \,:\, \nu_j < \gamma(\eta, P_t, P_{min}) + 2\}$$

Now, consider a station (say station S_1). Let A_1 be the access point to which this station is associated with. Let station S_1 belong to a hexagonal cell $d \in \mathcal{N}$, where $\mathcal{N}$ denotes the collection of cells deployed as in Figure 1. Now, we consider the hexagonal cell of radius $\nu_{j_0} \cdot R_t$ centred at cell d and try to compute the dependency probabilities of another station (S_2), when S_2 is present within the hexagon of radius $\nu_{j_0} \cdot R_t$ on the same channel as station S_1.

II. Probability of various dependencies

A. Probability of Type I dependency

In this section, we find the probability of type I dependency between two station S_1 and S_2. Let $p_j^{(1)}$ denote the probability that two stations have type I dependency, given that they belong to co-channels cells with centres D_j unit apart.

When the cells are partially dependent, to ensure type I dependency between S_1 and S_2, we require S_1 and S_2 to be in the interference range of each other. We also require S_1 and S_2 be outside the interference range access points A_2 and A_1 respectively.

Let $\mathcal{H}((x,y),R)$ denote a regular hexagon of radius R centred at (x,y). Let the positions of the access points A_1 and A_2 be (x_1,y_1) and (x_2,y_2) respectively. Now, let us define the following

$$\Delta_1^j = \{(x,y) \in \mathbb{R}^2 : (x,y) \in \mathcal{H}((x_1,y_1),R_t), (x,y) \notin \mathcal{H}((x_2,y_2),R_i),\ A_1 \text{ and } A_2 \text{ are in } j^{\text{th}} \text{ tier co-channel cells}\}$$
$$\Delta_2^j = \{(x,y) \in \mathbb{R}^2 : (x,y) \in \mathcal{H}((x_2,y_2),R_t), (x,y) \notin \mathcal{H}((x_1,y_1),R_i),\ A_1 \text{ and } A_2 \text{ are in } j^{\text{th}} \text{ tier co-channel cells}\}$$

i.e., Δ_1^j/Δ_2^j denotes the area outside the interference range of access point A_2/A_1 and within the hexagonal cell of radius R_t centred at access point A_1/A_2, when A_1 and A_2 are j^{th} tier co-channel cells. Let $p_1(x,y)$ denote the probability that S_1 is at $(x,y) \in \Delta_1^j$. Also, let $p_2(x,y)$ denote the probability that $S_2 \in \mathcal{H}((x,y),R_i) \cap \Delta_2^j$, given that S_1 is at (x,y). Now, we can compute the probability of type I dependency between stations S_1 and S_2 as

$$p_j^{(1)} = \int_{\Delta_1^j} p_1(x,y) \cdot p_2(x,y)\, dx\, dy \tag{3}$$

The stations are uniformly distributed within the regular hexagonal cell with radius R_t of their associated access points. Thus, we have $p_1(x,y) = \frac{1}{3\sqrt{3}R_t^2/2}$. Since station S_2 also has a uniform distribution within its associated hexagonal cell, we have

$$p_2(x,y) = \frac{Area(\mathcal{H}((x,y),R_i) \cap \Delta_2^j)}{3\sqrt{3}R_t^2/2}$$

where $Area(\mathcal{X}), \mathcal{X} \subset \mathbb{R}^2$ denotes the area of the region in $\mathbb{R}^2$ denoted by the set $\mathcal{X}$. Thus, integral (3) reduces to

$$p_j^{(1)} = \frac{4}{27R_t^4} \int_{\Delta_1^j} Area(\mathcal{H}((x,y),R_i) \cap \Delta_2^j)\, dx\, dy \tag{4}$$

Next, we consider various possible interference scenarios for j^{th} tier co-channel cells and compute p_j for each of the cases.

Case 1: $D_j - 2R_t \leq R_i < D_j - R_t$, or equivalently $\nu_j - 2 \leq \gamma(\eta, P_t, P_{min}) < \nu_j - 1$. In this scenario, we can rewrite integral (4) as

$$p_j^{(1)} = \frac{4}{27R_t^4} \int_{\Delta} Area(\mathcal{H}((x,y),R_i) \cap \mathcal{H}((x_2,y_2),R_t))\, dx\, dy \tag{5}$$

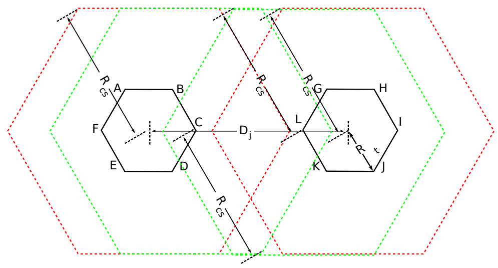

Fig. 2. Possible scenario for type I dependency under Case 1. Red and green hexagons show the interference range of APs and STAs respectively.

where $\Delta = \mathcal{H}((x_1, y_1), R_t) \cap \mathcal{H}((x_2 - R_t, y_2), R_t)$. After some geometric constructions, integral (5) becomes

$$p_j^{(1)} = \frac{2}{9\sqrt{3}R_t^4} \int_0^{R_i+2R_t-D_j} \int_0^{R_i+2R_t-D_j} xy \, dx \, dy = \frac{1}{18\sqrt{3}} \cdot \left(\frac{R_i - D_j + 2R_t}{R_t} \right)^4$$

Substituting for R_i and D_j in terms of R_t, we obtain

$$p_j^{(1)} = \frac{1}{18\sqrt{3}} \cdot (\gamma(\eta, P_t, P_{min}) + 2 - \nu_j)^4$$

Case 2: $D_j - R_t \le R_i < D_j$, or equivalently $\nu_j - 1 \le \gamma(\eta, P_t, P_{min}) < \nu_j$

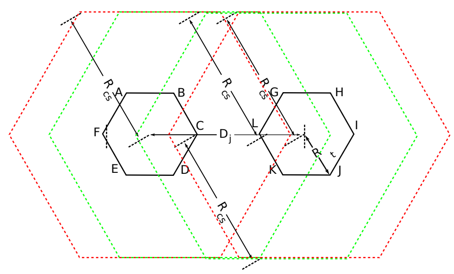

Fig. 3. Possible scenario for type I dependency under Case 2. Red and green hexagons show the interference range of APs and STAs respectively.

For this case, we evaluate integral (4) by splitting Δ_1^j and $\mathcal{H}((x, y), R_i) \cap \Delta_2^j$ into non-overlapping area. After some inferences based on geometry and calculus, we get

$$p_j^{(1)} = \frac{1}{9\sqrt{3}} \cdot \left(1 - \left(\frac{R_i - D_j + R_t}{R_t} \right)^4 - \frac{1}{2} \cdot \left(\frac{D_j - R_i}{R_t} \right)^4 \right) + \frac{1}{54} \cdot \left(\frac{R_i - D_j + R_t}{R_t} \right)^4$$

Substituting for R_i and D_j in terms of R_t, we obtain

$$p_j^{(1)} = \frac{1}{9\sqrt{3}} \cdot \left(1 - (\gamma(\eta, P_t, P_{min}) + 1 - \nu_j)^4 - \frac{1}{2} \cdot (\nu_j - \gamma(\eta, P_t, P_{min}))^4 \right) + \frac{1}{54} \cdot (\gamma(\eta, P_t, P_{min}) + 1 - \nu_j)^4$$

Case 3: $D_j \le R_i$, or equivalently $\nu_j \le \gamma(\eta, P_t, P_{min})$. In this case, the access points are within interference range of each other. Thus, in this scenario, it is impossible to have just *STA-STA* dependency between stations S_1 and S_2. Thus, we have $p_j^{(1)} = 0$

B. Probability of Type II dependency

In this section, we find the probability of type II dependency between two station S_1 and S_2. Let $p_j^{(2)}$ denote the probability that two stations have type II dependency, given that they belong to co-channels cells with centres D_j unit apart. The computation of type II dependence probability can be split into two simple cases as below.

Case 1: $D_j - 2R_t \le R_i < D_j$, or equivalently $\nu_j - 2 \le \gamma(\eta, P_t, P_{min}) < \nu_j$. In this case, the co-channel access points are out of each others interference range. Thus, $p_j^{(2)} = 0$

Case 2: $D_j \le R_i$, or equivalently $\nu_j \le \gamma(\eta, P_t, P_{min})$. In this case, the co-channel access points are within each others interference range. Thus, $p_j^{(2)} = 1$

C. Probability of Type III dependency

In this section, we find the probability of type III dependency between two station S_1 and S_2. Let $p_j^{(3)}$ denote the probability that two stations have type III dependency, given that they belong to co-channels cells with centres D_j unit apart.

Case 1: $D_j - 2R_t \le R_i < D_j - R_t$, or equivalently $\nu_j - 2 \le \gamma(\eta, P_t, P_{min}) < \nu_j - 1$. In this case, the interference region of the access points do no over lap the j^{th} tier co-channel cells. Thus, in this case, type III dependency cannot occur i.e., $p_j^{(3)} = 0$

Case 2: $D_j - R_t \le R_i < D_j$, or equivalently $\nu_j - 1 \le \gamma(\eta, P_t, P_{min}) < \nu_j$. For this case, we have

$$p_j^{(3)} = 1 - P[\text{Access point } A_1 \text{ does not interfere with station } S_2 \text{ and access point } A_2 \text{ does interferes with station } S_1]$$

From application of simple geometric argument, we get

$$p_j^{(3)} = 1 - \left(1 - \frac{\sqrt{3}\cdot(R_i + R_t - D_j)^2/2}{3\sqrt{3}R_t^2/2}\right)^2 = 1 - \left(1 - \frac{1}{3}\cdot(\gamma(\eta, P_t, P_{min}) + 1 - \nu_j)^2\right)^2$$

Case 3: $D_j \le R_i$, or equivalently $\nu_j \le \gamma(\eta, P_t, P_{min})$. In this case, the access points interfere with each other. Thus, we do not have type III dependency i.e., $p_j^{(3)} = 0$

D. Final probability expression for each type of dependency

Let $\mathcal{N}_j \subset \mathcal{N}$ denote the j^{th} tier co-channel cells of cell d and let n_j be its cardinality. Let $\mathcal{N}' = \cup_{j=1}^{j_0}\mathcal{N}_j$. Since elements of the set $\{\mathcal{N}_j, j_0 \ge j \ge 1\}$ do not overlap with each other, we have $|\mathcal{N}'| = \sum_{j_0 \ge j \ge 1} n_j$. Let us define an indicator variable as follows:

$$I^j(S_2) = \begin{cases} 1 & \text{if station } S_2 \text{ is in a } j^{\text{th}} \text{ tier co-channel cell} \\ 0 & \text{otherwise} \end{cases}$$

Let $E^{(i)}$ and $p^{(i)}$ denote the event and probability of type i dependency between stations S_1 and S_2 given that $S_2 \in \mathcal{N}'$. Then, we have

$$p^{(i)} = P[E^{(i)}|S_2 \in \mathcal{N}'] = \sum_{l\in\mathcal{N}'} P[S_2 \in Cell\, l, E^{(i)}|S_2 \in \mathcal{N}'] = \sum_{l\in\mathcal{N}'}\sum_{j_0\ge j\ge 1} P[S_2 \in Cell\, l, E^{(i)}, I^j(S_2) = 1|S_2 \in \mathcal{N}']$$

By conditioning on the events and simplifying, we get

$$p^{(1)} = \sum_{l\in\mathcal{N}'}\sum_{j_0\ge j\ge 1} P[S_2 \in Cell\, l|S_2 \in \mathcal{N}'] \cdot P[I^j(S_2) = 1|S_2 \in Cell\, l] \cdot P[E^{(i)}|I^j(S_2) = 1, S_2 \in Cell\, l] \tag{6}$$

By definition, we have

$$P[E^{(i)}|I^j(S_2) = 1, S_2 \in Cell\, l] = p_j^{(i)} \quad \forall l \in \mathcal{N}'$$

Station S_2 has a uniform distribution over the deployed area, we have $P[S_2 \in Cell\, l|S_2 \in \mathcal{N}'] = \frac{1}{|\mathcal{N}'|}, \forall l \in \mathcal{N}'$. We also have

$$P[I^j(S_2) = 1|S_2 \in Cell\, l] = \begin{cases} 1 & \text{if } S_2 \in \mathcal{N}_j \\ 0 & \text{otherwise} \end{cases}$$

Plugging in the various probabilities into Equation (6), we obtain

$$p^{(i)} = \sum_{j_0\ge j\ge 1} p_j^{(i)} \cdot P[S_2 \in \mathcal{N}_j|S_2 \in \mathcal{N}'] = \left(\frac{1}{\sum_{j_0\ge j\ge 1} n_j}\right) \cdot \sum_{j_0\ge j\ge 1} p_j^{(i)} \cdot n_j$$

where $n_j \in \{6, 12\}$ is the number of j^{th} tier co-channel cells of cell d.

TABLE I

TABLE SHOWING THE VALUES OF p AGAINST VARIOUS VALUES OF r_t, GIVEN THAT $\alpha = 1$ AND $\eta = 3.5$

r_t	P_t	$p^{(1)}$	$p^{(2)}$	$p^{(3)}$
54	-44	0.0010	0.9510	0.0112
48	-60	0.0074	0.7778	0.0112
36	-69	0.0076	0.7143	0.0000
24	-73	0.0000	0.6000	0.0000
12	-85	0.0416	0.0000	0.3686
1	-90	0.0321	0.0000	0.0000